Beskrivning av kaströrelse <u>utan</u> luftmotstånd.

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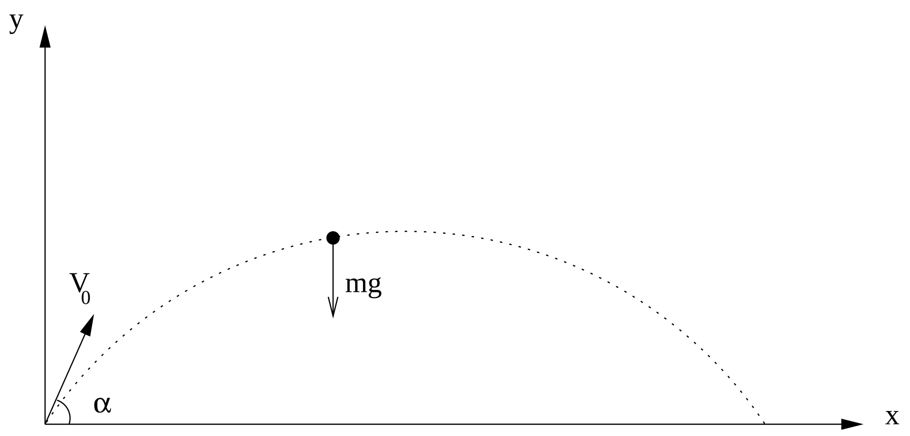


$\rightarrow$ x: m$\ddot{x} = \Sigma F_x = 0 \Rightarrow \ddot{x} = 0$ (1)

$\uparrow$ y: m$\ddot{y} = \Sigma F_y = -mg \Rightarrow \ddot{y} = -g$ (2)

<u>BV:</u>

$$\begin{aligned} x(0) &= 0 \\ \dot{x}(0) &= v_0 \cos\alpha \\ y(0) &= 0 \\ \dot{y}(0) &= v_0 \sin\alpha \end{aligned}$$

Integrera (1) map t $\Rightarrow \dot{x} = \text{const} = v_0 \cos\alpha$ (3)

Integrera (3) map t $\Rightarrow x = v_0 \cos\alpha \cdot t + C$ (C $= v_0 = 0$) (4)

Integrera (2) map t $\Rightarrow \dot{y} = -gt + v_0 \sin\alpha$ (5)

Integrera (5) map t $\Rightarrow y = -\frac{1}{2}gt^2 + v_0 \sin\alpha \cdot t + C$ (C $= v_0 = 0$) (6)

a) Sök bankurvans ekvation $y = y(x)$.

Enligt ekv(4) är: $t = \frac{x}{v_0 \cos\alpha}$ in i ekv(6) $\Rightarrow$

$$y = -\frac{g}{2} \cdot \frac{x^2}{v_0^2 \cos^2\alpha} + v_0 \sin\alpha \cdot \frac{x}{v_0 \cos\alpha} =$$

$$y = -\frac{gx^2}{2v_0^2 \cos^2\alpha} + x \tan\alpha \ (7)$$

Ekvation (7) brukar kallas för "kastparabelns ekvation".

b) Bestäm kurvans högsta höjd h (fås då $\dot{y} = 0$)

$$\dot{y} = -gt + v_0 \sin\alpha = 0 \Rightarrow t_* = \frac{v_o \sin\alpha}{g}$$

t_* in i ekv(6) ger:

$$h = y(t_*) = -\frac{g}{2} \cdot \frac{v_0^2 \sin^2\alpha}{g^2} + v_0 \sin\alpha \cdot \frac{v_0 \sin\alpha}{g} =$$

$$h = \frac{v_0^2 \sin^2\alpha}{2g}$$

c) Bestäm kastvidden L (fås ur kastparabeln då $y = 0$)

$$0 = -\frac{gL^2}{2v_0^2 \cos^2\alpha} + L \tan\alpha \Rightarrow L(\tan\alpha - \frac{gL}{2v_0^2 \cos^2\alpha}) = 0$$

$$L = \frac{2v_0^2 \cos^2\alpha \cdot \tan\alpha}{g} = \frac{2v_0^2 \cdot \sin\alpha \cos\alpha}{g} = \frac{v_0^2}{g} \cdot \sin(2\alpha)$$

$$L_{max} = \frac{v_0^2}{g}$$

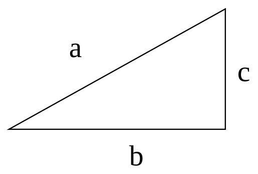
d) Bestäm α för att "träffa" den givna punkten (x_1, y_1).

Punkten skall uppfylla ekv(7).

$y_1 = -\frac{gx_1^2}{2v_0^2 \cos\alpha} + x_1 \tan\alpha =$

$y_1 \cdot 2v_0^2 \cos\alpha = -gx_1^2 + 2v_0^2 \cdot x_1 \tan\alpha \cdot \cos^2\alpha$

Gör följande variabelsubstitution:



där $a = \sqrt{1+\gamma^2}, b = 1, c = \gamma \Rightarrow$

$\sin\alpha = \frac{\gamma}{\sqrt{1+\gamma^2}}$, $\cos\alpha = \frac{1}{\sqrt{1+\gamma^2}}$

$y_1 \cdot 2v_0^2 \cdot \frac{1}{(1+\gamma^2)} = -gx_1^2 + x_1 2v_0^2 \cdot \frac{\gamma}{(1+\gamma^2)} =$

$y_1 \cdot 2v_0^2 = -gx_1^2(1+\gamma^2) + x_1 2v_0 \cdot \gamma$

$\gamma_{1,2} = \frac{v_0^2}{gx_1} \pm \sqrt{(\frac{v_0^2}{gx_1})^2 - \frac{2v_0^2 y_1}{gx_1^2} - 1} \Rightarrow$

$\alpha = \arctan\gamma$